Cutoff Frequencies of Guiding Structures with Circular and Planar Boundaries

B. N. Das, S. B. Chakrabarty, and A. K. Mallick, *Member, IEEE*

Abstract—**The paper presents a method of evaluation of cutoff frequencies of a guiding structure consisting of two eccentric circular arcs with edges shorted by conducting planes. Combination of conformal transformation and method of finite difference is used for the analysis. The general formulation is applied to the limiting cases of lunar guide and also the guiding structure of semicircular cross section with a semicircular dent along its diameter. Numerical data for TE and TM modes are presented for different angular separation between the shorting planes, distance between the centers and ratio of radii.**

I. Introduction

Kuttler [1] evaluated cutoff frequencies of a lunar guide which is an eccentric coaxial line with a shorting conductor connecting the closest points along the line joining their centers. Kuttler [1] used the combination of conformal transformation and method of intermediate problems for solving the weighted Helmholtz equation resulting from conformal transformation. It is worthwhile to study the cutoff frequencies when the shorting conductor between the two cylinders is split into two parts and rotated.

In view of the complexity of the method of intermediate problems from a conceptual and computational point of view, the weighted Helmholtz equation is solved using the method of finite difference [2].

Numerical data on cutoff frequencies of TE and TM modes for different angular separation between the shorting planes and different distance between the centers of the circular arc with ratio of their radii as parameter are presented.

II. Analysis

Consider the structure shown in Fig. 1. Application of the conformal transformation [1], [3] transforms the structure of Fig. 1 to a rectangle with conducting boundaries as shown in Fig. 2. All mathematical expressions including conformal transformation are derived in terms of absolute geometrical parameters. The separation between the conducting boundaries in Fig. 2 along the x and y axes is $x_2 - x_1$ [1] and $y_2 - y_1$

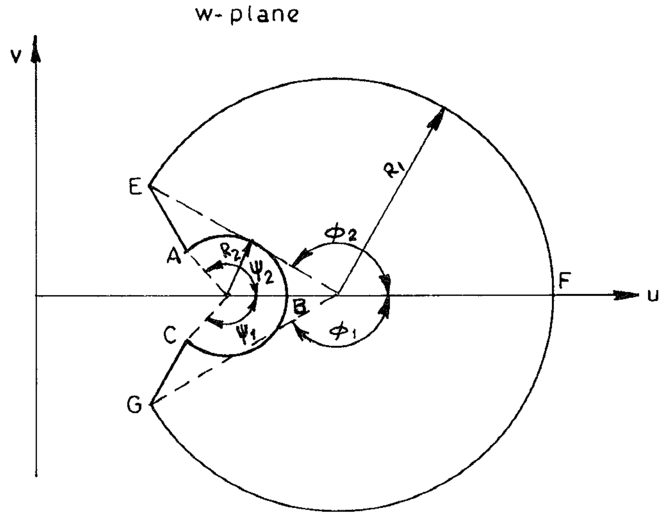

Fig. 1. w-plane representation of guiding structure with circular and planar boundaries.

respectively where

$$x_2 - x_1 = \cosh^{-1} \frac{1 - (\frac{R_2}{R_1})^2 - (\frac{D}{R_1})^2}{2\frac{R_2 D}{R_1^2}} - \cosh^{-1} \frac{1 - (\frac{R_2}{R_1})^2 + (\frac{D}{R_1})^2}{2\frac{D}{R_1}} \tag{1}$$

and

$$y_2 - y_1 = \begin{cases} \tan^{-1} \frac{\sinh x_1 \sin \phi_1}{2+\cosh x_1 (1+\cos \phi_1)} - \tan^{-1} \frac{\sinh x_1 \sin \phi_2}{2+\cosh x_1 (1+\cos \phi_2)} \\ \tan^{-1} \frac{\sinh x_2 \sin \psi_1}{2+\cosh x_2 (1+\cos \psi_1)} - \tan^{-1} \frac{\sinh x_2 \sin \psi_2}{2+\cosh x_2 (1+\cos \psi_2)} \end{cases}. \tag{2}$$

The values of ϕ and ψ are so selected that the values of y along $x = x_1$ and $x = x_2$ are identical. The weighted Helmholtz equation resulting from the conformal transformation assumes the form [1]

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + k^2 R_1^2 \left\{ \frac{\sinh(x_1)}{\cosh(x) - \cos(y)} \right\}^2 U = 0. \tag{3}$$

The dimensionless parameter kR_1 is obtained from the solution of simultaneous equations resulting from the application of the method of finite difference. The rectangular boundary in the transformed plane is divided into rectangular grid as shown in Fig. 2. If M and N are the number of nodes along

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The authors are with the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur-721302, India.

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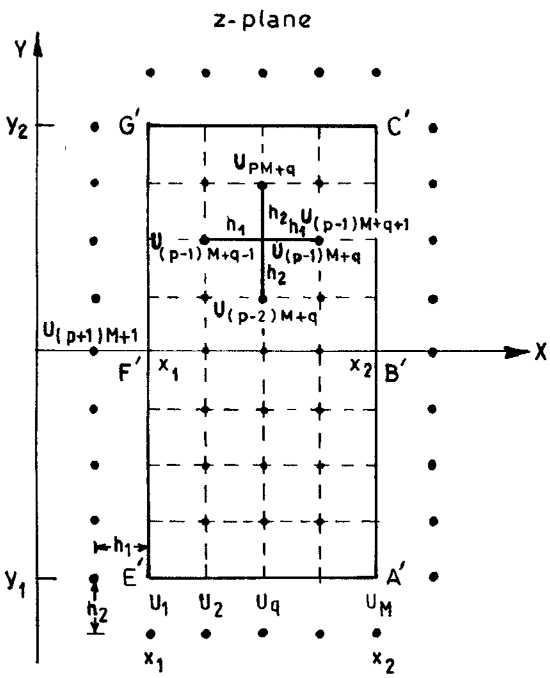

Fig. 2. z-plane representation of the structure of Fig. 1 obtained through conformal transformation and the geometry of grid structure.

the x and y directions respectively, the separation between the nodes in the two directions are

$$h_1 = \frac{x_2 - x_1}{M}, \quad h_2 = \frac{y_2 - y_1}{N}. \tag{4}$$

At the nodes of Fig. 2, the unknown function U are represented by $U_{(p-1)M+q}$ where $1 \le p \le N$ and $1 \le q \le M$.

Following the procedure suggested in the literature [2], the difference equation reduces to the form

$$\begin{aligned} &- U_{pM+q}h_1^2 - U_{(p-2)M+q}h_1^2 - U_{(p-1)M+q+1}h_2^2 \\ &- U_{(p-1)M+q-1}h_2^2 + \Bigg[(2h_1^2 + 2h_2^2) \\ &- k^2h_1^2h_2^2R_1^2\left\{\frac{\sinh x_1}{\cosh(x_q) - \cos(y_p)}\right\}^2\Bigg]U_{(p-1)M+q} \\ &= 0 \end{aligned} \tag{5}$$

where

$$\begin{aligned} x_q &= x_1 + (q-1)h_1 \\ y_p &= y_1 + (p-1)h_2 \end{aligned}.$$

Equation (5) gives a set of simultaneous equations as p and q assumes values over the range $1 \le p \le N$ and $1 \le q \le M$ as stated above. This set of simultaneous equations leads to a matrix equation of the form

$$\big([A] - k^2R_1^2h_1^2h_2^2[B]\big)[U] = [0]. \tag{6}$$

In (6), $[A]$ is a square matrix whose diagonal elements are $2(h_1^2 + h_2^2)$ and nondiagonal elements are 0's, $-h_1^2$ and $-h_2^2$ as dictated by (8) and the boundary conditions. $[B]$ is a diagonal matrix whose diagonal elements are the derivative of $|\frac{dw}{dz}|$ at x_q and y_p. $[U]$ is a column matrix and $[0]$ is a null column matrix. Order of all these matrices are $(M-p)(N-q)$ where p and q assume values of 0 or 2, depending on the boundary conditions. Representing the eigenvalue of (6) as

$$\xi = k^2R_1^2h_1^2h_2^2 \tag{7}$$

the matrix equation assumes the form

$$([A] - \xi[B])[U] = [0] \tag{8}$$

The eigenvalues ξ are found from the characteristic equation

$$\det([A] - \xi[B]) = 0. \tag{9}$$

The boundary conditions are 1) Dirichlet boundary condition

$$U = 0 \tag{10}$$

for TM mode and 2) Neumann boundary condition

$$\frac{\partial U}{\partial n} = 0 \tag{11}$$

for TE mode.

Following the method suggested in the literature [2] for forming the equations in accordance with Dirichlet and Neumann boundary conditions, a set of simultaneous equations in terms of unknown potentials at the nodes are obtained.

TABLE I
TM FREQUENCIES k_j THE LINE OF FIG. 1

j	$\frac{R_2}{R_1} = 0.5, \frac{D}{R_1} = 0.1$			$\frac{R_2}{R_1} = 0.25, \frac{D}{R_1} = 0.25$		
	$\phi_1 = 0$, $\phi_2 = \pi$	$\phi_1 = \frac{5\pi}{6}$, $\phi_2 = -\frac{5\pi}{6}$	$\phi_1 = -\pi$, $\phi_2 = \pi$	$\phi_1 = 0$, $\phi_2 = \pi$	$\phi_1 = \frac{5\pi}{6}$, $\phi_2 = -\frac{5\pi}{6}$	$\phi_1 = -\pi$, $\phi_2 = \pi$
1	5.95	5.43	5.42	4.24	3.46	3.44
2	6.87	5.95	5.94	5.50	4.27	4.23
3	7.66	6.44	6.42	6.57	4.98	4.88
4	8.42	6.89	6.86	7.57	5.66	5.42
5	9.27	7.32	7.26	7.62	6.33	5.86
6	10.19	7.76	7.65	8.66	6.62	6.23
7	10.98	8.21	8.02	9.09	6.99	6.52
8	11.16	8.69	8.41	9.69	7.57	6.63
9	12.03	9.19	8.82	10.33	7.65	7.07
10	12.14	9.72	9.25	10.58	8.31	7.46

III. NUMERICAL RESULT AND DISCUSSION

Using (5) and the boundary conditions (10) and (11), the matrices $[A]$ $[B]$ are evaluated for different values of $\frac{D}{R_1}$ and $\frac{D}{R_2}$. Substituting these matrices in (9), the eigenvalues and hence cutoff frequencies for TE and TM modes are determined for the case of a lunar guide for which $\phi_1 = -\pi$ and $\phi_2 = \pi$

TABLE II
TE FREQUENCIES k_j THE LINE OF FIG 1

j	$\frac{R_2}{R_1}=0.5, \frac{D}{R_1}=0.1$			$\frac{R_2}{R_1}=0.25, \frac{D}{R_1}=0.25$		
	$\phi_1=0$ $\phi_2=\pi$	$\phi_1=\frac{5\pi}{6}$ $\phi_2=-\frac{5\pi}{6}$	$\phi_1=-\pi$ $\phi_2=\pi$	$\phi_1=0$ $\phi_2=\pi$	$\phi_1=\frac{5\pi}{6}$ $\phi_2=-\frac{5\pi}{6}$	$\phi_1=-\pi$ $\phi_2=\pi$
2	1.36	0.89	0.75	1.67	1.22	1.02
3	2.67	1.66	1.36	2.93	2.00	1.66
4	3.92	2.44	2.02	3.93	2.74	2.32
5	5.10	3.21	2.67	4.08	3.42	2.91
6	5.58	3.94	3.30	5.14	3.92	3.48
7	6.22	4.65	3.91	5.68	4.08	3.89
8	6.69	5.33	4.51	6.14	4.71	4.04
9	7.29	5.55	5.08	6.57	4.91	4.55
10	7.55	5.98	5.55	6.96	5.32	4.94

for the purpose of verification of the method. The deviation of the results with those found by Kuttler using the method of intermediate problems is below 1%. This agreement justifies the validity of the analysis.

Computation is also carried out for $\phi_1 = 0$, $\phi_2 = \pi$ and $\phi_1 = -\frac{5\pi}{6}$, $\phi_2 = \frac{5\pi}{6}$ with $\frac{R_2}{R_1} = 0.5$, $\frac{D}{R_1} = 0.1$ and $\frac{R_2}{R_1} = 0.25$, $\frac{D}{R_1} = 0.25$. Using (2) (ϕ_1, ϕ_2) and (ψ_1, ψ_2) for which the transformed structure is a rectangle is found. The results converge for $M = 7$, $N = 40$. The numerical data on cutoff frequencies TE and TM modes are presented in Tables I and II. Results presented reveal that with the changes in shape of structure from lunar to semicircular shape the change in dominant TM cutoff frequency is about 9% and that in dominant TE is around 56%.

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